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Formulas and constants for the calculation of the Swiss conformal cylindrical projection and for the transformation between coordinate systems

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1 Basic Information

1.1 Summary of the reference systems and reference frames used in Switzerland

System	Frame	Ellipsoid	Map projection
ETRS89	ETRF93	GRS80	(UTM)
CHTRS95	CHTRFxx	GRS80	(UTM, Zone 32)
CH1903	LV03 (LV03-C, LV03-M)	Bessel 1841	oblique conformal cylindrical
CH1903+	LV95	Bessel 1841	oblique conformal cylindrical

The 3D-reference system **CHTRS95** (Swiss Terrestrial Reference System 1995) is tied closely to the European Terrestrial Reference System **ETRS89** and is identical to it at the epoch 1993.0. Because, until now there are no reasons to change this, the 2 systems will remain identical for some time. CHTRF95, CHTRF98, CHTRF2004, CHTRF2010 and CHTRF2016, the reference frames realised until now, are based on the geocentric-Cartesian coordinates of the fundamental station in Zimmerwald in ETRF93 at the epoch 1993.0.

CH1903 is the classical local reference system derived from triangulation. Its reference frame **LV03** (Landesvermessung 1903) was the official frame for cadastral surveying in most of the cantons until the end of 2016. Originally it was defined only locally with respect to the observatory of Berne and with the plane coordinates 0 / 0 for the projection centre. These so-called "civilian coordinates" (LV03-C) were later replaced by the "military coordinates" (LV03-M) where the projection centre got a false easting of 600'000 m and a false northing of 200'000 m. This choice avoided negative numbers and confusion between easting and northing (easting is always larger than northing) everywhere in Switzerland. Furthermore, the projection was defined more globally with respect to the Greenwich meridian (instead of the meridian of Berne). During the 20th century all cantons adopted LV03-M. Technically the two systems LV03-C and LV03-M can be calculated with exactly the same formulas and constants except for the false easting/northing. In LV03, the easting is usually abbreviated by the letter y, whereas the northing gets the letter x.

The local reference system **CH1903+** with its reference frame **LV95** (Landesvermessung 1995) is derived from CHTRS95 and is the official system for national and cadastral survey since 2016. In defining CH1903+, it was important that it remains as close as possible to the old reference system CH1903. The parameters defining the system were transferred from the old fundamental station (old observatory of Bern, which does not exist anymore) to the new fundamental station in Zimmerwald. To avoid confusion between LV03 and LV95, the projection centre (which remained at the observatory of Berne) got a false easting/northing of 2'600'000 / 1'200'000 m. Except from this offset all the calculation formulas and constants remain the same as in CH1903/LV03. In LV95, the easting is usually abbreviated by the letter E, whereas the northing gets the letter N.

The reference frames LV03 and LV95 show differences of up to 1.6 metres because of the local distortions of LV03. These local distortions are modelled with a local affine transformation (program FINELTRA, data set CHENyx06) or the derived distortion grids in various formats for GIS software or GPS receivers.

All transformations between the reference systems and frames used in Switzerland are possible with the software REFRAME, which is also available as a free internet service.

1.2 Height systems used in Switzerland

The official height system **LN02**, which is still in use, was defined in 1902 by fixing the 'height above sea level' of the Repère Pierre du Niton H(RPN)=373.6 m in Geneva, which was obtained from a connection measurement to the tide gauge in Marseilles. The heights of the levelling bench marks were determined by pure levelling. The relative heights of the nodal points of the 'nivellement de précision' (1864 - 1891) were kept fixed, and the gravity field was not taken into consideration.

The new height system **LHN95** (Landeshöhennetz 1995) is also based on the height of the RPN. However, in this height system the derived geopotential number of the fundamental station in Zimmerwald was declared as the defining constant. The heights of the bench marks of LHN95 are calculated in a kinematic adjustment of the levelling network which takes gravity measurements into account. The user obtains orthometric heights (LHN95-o) derived from the calculated geopotential numbers, but normal heights (LHN95-n) are available as well.

For exchanging data with neighbouring countries, an additional height system **CHVN95** was defined. It is, for the time being, identical to the European vertical height system EVRS. It is based on the height definition of the tide gauge in Amsterdam (NAP) and on the results of the European levelling network (UELN) and the European Vertical Reference Network (EUVN). In this system, height information is exchanged in the form of geopotential numbers or normal heights.

The relationship between the orthometric heights of LHN95 with the ellipsoidal heights of CH1903+ and CHTRS95 is guaranteed through the official Swiss geoid model **CHGeo2004**, which was calculated out of gravity, deflections of the vertical and GPS/levelling.

The differences between the normal heights of LHN95 and CHVN95 (~EVRS) are modelled preliminarily by a simple offset of 10.3 cm (LHN95 height minus UELN height). This value results from a comparison of the results of LHN95 with the values of UELN95/98.

The differences between LHN95 (orthometric) and LN02 are between -20 cm in the north of the country and +50 cm on the highest peaks of the Alps and show a very strong correlation with height. They cannot be modelled by a single offset because of their different way of gravity reduction, the treatment of vertical movements and the constraints introduced in LN02. For a transformation between these height systems, the differences are separated in two parts: The first part describes the differences between LN02 and normal heights, whereas the second part describes a height scale factor which is used for a transformation between normal heights and orthometric heights. Both parts are stored in grids with a resolution of 1 km.

All the height transformations for Switzerland are possible with the software HTRANS or with the more general program REFRAME.

1.3 Reference ellipsoids used in Switzerland

Ellipsoid	Semi-major axis a [m]	Semi-minor axis b [m]	Flattening 1/f	1st num. eccentr. e^2
Bessel 1841	6377397.155	6356078.962822	299.15281285	0.006674372230614
GRS 80	6378137.000	6356752.314140	298.257222101	0.006694380023011
WGS 84	6378137.000	6356752.314245	298.257223563	0.006694379990197

Flattening: $f = \frac{a - b}{a}$

first numerical eccentricity squared: $e^2 = \frac{a^2 - b^2}{a^2}$

1.4 Transformation parameters CHTRS95/ETRS89 ⇔ CH1903+

These parameters have been used since 1997 for the transformation between CHTRS95 and CH1903+. Without any restrictions they can also be used for the system ETRS89 and for many applications also for CH1903. But in the case of CH1903, one must be aware that because of the local distortions of this network the transformed coordinates can be false by up to 1.6 meters compared to the official coordinates in CH1903.

$$X_{CH1903+} = X_{CHTRS95} - 674.374 \text{ m}$$
$$Y_{CH1903+} = Y_{CHTRS95} - 15.056 \text{ m}$$
$$Z_{CH1903+} = Z_{CHTRS95} - 405.346 \text{ m}$$

1.5 Granit87 parameters

These parameters were used between 1987 and 1997 for the transformation between CH1903 and WGS84. We do not recommend their use anymore.

dX = 660.077 m
dY = 13.551 m
dZ = 369.344 m
s = 1.00000566 (m = 5.66 ppm)

$\alpha = r_x$ = 2.484 cc (centesimal seconds)
$\beta = r_y$ = 1.783 cc (centesimal seconds)
$\gamma = r_z$ = 2.939 cc (centesimal seconds)

to be used with the transformation formulas:

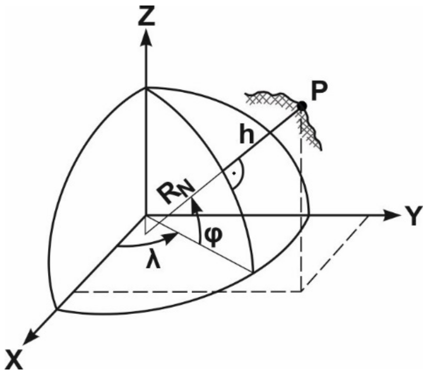
$$\begin{pmatrix} X_{WGS84} \\ Y_{WGS84} \\ Z_{WGS84} \end{pmatrix} = \begin{pmatrix} dX \\ dY \\ dZ \end{pmatrix} + s \cdot D \cdot \begin{pmatrix} X_{CH1903} \\ Y_{CH1903} \\ Z_{CH1903} \end{pmatrix}$$

with rotation matrix $D = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$ and its elements

$r_{11} = \cos\beta \cos\gamma$
$r_{21} = -\cos\beta \sin\gamma$
$r_{31} = \sin\beta$
$r_{12} = \cos\alpha \sin\gamma + \sin\alpha \sin\beta \cos\gamma$
$r_{22} = \cos\alpha \cos\gamma - \sin\alpha \sin\beta \sin\gamma$
$r_{32} = -\sin\alpha \cos\beta$
$r_{13} = \sin\alpha \sin\gamma - \cos\alpha \sin\beta \cos\gamma$
$r_{23} = \sin\alpha \cos\gamma + \cos\alpha \sin\beta \sin\gamma$
$r_{33} = \cos\alpha \cos\beta$

2 Conversion between ellipsoidal and geocentric-Cartesian coordinates

2.1 Ellipsoidal coordinates (longitude λ, latitude φ, height h) ⇒ geocentric-Cartesian coordinates X, Y, Z



$$X = (R_N + h) \cdot \cos\varphi \cdot \cos\lambda$$
$$Y = (R_N + h) \cdot \cos\varphi \cdot \sin\lambda$$
$$Z = \left(R_N \cdot (1 - e^2) + h\right) \cdot \sin\varphi$$

with normal radius of curvature: $R_N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}}$

The parameters a and e are dependent on the reference ellipsoid:

a = semi-major axis of the reference ellipsoid
b = semi-minor axis of the reference ellipsoid

e = first numerical eccentricity of the ellipsoid = $\frac{\sqrt{a^2 - b^2}}{a}$

2.2 Geocentric-Cartesian coordinates X, Y, Z ⇒ ellipsoidal coordinates (longitude λ, latitude φ, height h)

$$\lambda = \arctan\left(\frac{Y}{X}\right) \qquad \varphi = \arctan\left(\frac{\frac{Z}{\sqrt{X^2 + Y^2}}}{1 - \frac{R_N \cdot e^2}{R_N + h}}\right) \qquad h = \frac{\sqrt{X^2 + Y^2}}{\cos\varphi} - R_N$$

with $R_N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}}$

Please note: The quantities φ, R_N and h are dependent on each other. Therefore, they have to be calculated by iteration (starting with an approximate value φ_0):

Proposed value for φ_0: $\varphi_0 = \arctan \frac{Z}{\sqrt{X^2 + Y^2}}$

3 Swiss projection formulas

3.1 Notation, constants, auxiliary values

Notation

φ, λ: ellipsoidal latitude and longitude in the system CH1903/CH1903+ relative to Greenwich
b, l : spherical coordinates relative to Bern
$\bar{b}, \bar{l}$: spherical coordinates relative to the pseudo-equatorial system in Bern
Y, X : civilian projection coordinates (LV03-C or LV95-C)
y, x : official (military) projection coordinates in LV03
E, N : official projection coordinates in LV95

Where not indicated, the units for angles are radians [rad] and the units for lengths are meters [m] in all formulas.

Constants

a	=	6377397.155 m	semi-major axis of the Bessel-ellipsoid
E^2	=	0.006674372230614	1st numerical eccentricity (squared) of the Bessel ellipsoid (*)
φ_0	=	46° 57' 08.66"	ellipsoidal latitude of the projection centre in Bern (**)
λ_0	=	7° 26' 22.50"	ellipsoidal longitude of the projection centre in Bern (**)

(*) In order to distinguish it from the Euler constant e, the 1st numerical eccentricity in these formulas is noted as E. Not to be confused with the Easting in LV95.

(**) These are the so-called 'old values', which are still valid for all geodetic purposes. The so-called 'new values' (from a new determination of the astronomical coordinates of the fundamental station in Bern from 1938: φ_0 = 46° 57' 07.89", λ_0 = 7° 26' 22.335") have only been used for cartographic purposes (indication of latitudes and longitudes on the national maps). We do not recommend the use of these values.

Calculation of auxiliary values

Radius of the projection sphere:

$$R = \frac{a \cdot \sqrt{1-E^2}}{1-E^2 \sin^2 \varphi_0} \qquad = 6378815.90365 \text{ m}$$

Relat. between longitude on sphere and on ellipsoid:

$$\alpha = \sqrt{1 + \frac{E^2}{1-E^2} \cdot \cos^4 \varphi_0} \qquad = 1.00072913843038$$

Latitude of the fundamental point on the sphere:

$$b_0 = \arcsin\left(\frac{\sin \varphi_0}{\alpha}\right) \qquad = 46° 54' 27.83324844"$$

Constant of the latitude formula:

$$K = \ln\left(\tan\left(\frac{\pi}{4} + \frac{b_0}{2}\right)\right) - \alpha \cdot \ln\left(\tan\left(\frac{\pi}{4} + \frac{\varphi_0}{2}\right)\right) + \frac{\alpha \cdot E}{2} \cdot \ln\left(\frac{1+E \cdot \sin \varphi_0}{1-E \cdot \sin \varphi_0}\right) \qquad = 0.0030667323772751$$

3.2 Ellipsoidal coordinates (λ, φ on the Bessel ellipsoid) ⇒ Swiss projection coordinates (rigorous formulas)

The numerical calculation is performed for the station Rigi with the following values:

φ = 47° 03' 28.95659233" = 0.821317799 rad

λ = 8° 29' 11.11127154" = 0.148115967 rad

a) Bessel ellipsoid (φ, λ) ⇒ sphere (b, l) (Gauss projection)

Auxiliary value:

$$S = \alpha \cdot \ln\left(\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\right) - \frac{\alpha \cdot E}{2} \cdot \ln\left(\frac{1 + E \cdot \sin\varphi}{1 - E \cdot \sin\varphi}\right) + K$$

= 0.931969601072417

spherical latitude:

$$b = 2 \cdot (\arctan\left(e^{S}\right) - \frac{\pi}{4})$$

= 0.820535226 rad
(= 47° 00' 47.539422864")

spherical longitude:

$$l = \alpha \cdot (\lambda - \lambda_0)$$

= 0.0182840649 rad
(= 1° 02' 51.3591108468")

b) equator system (b, l) ⇒ pseudo-equator system ($\bar{b}$, $\bar{l}$) (rotation)

$$\bar{l} = \arctan\left(\frac{\sin l}{\sin b_0 \cdot \tan b + \cos b_0 \cdot \cos l}\right)$$

= 0.0124662714 rad
(= 0° 42' 51.3530463924")

$$\bar{b} = \arcsin\left(\cos b_0 \cdot \sin b - \sin b_0 \cdot \cos b \cdot \cos l\right)$$

= 0.00192409259 rad
(= 0° 06' 36.8725855284")

c) sphere ($\bar{b}$, $\bar{l}$) ⇒ projection plane (y, x) (Mercator projection)

$$Y = R \cdot \bar{l}$$

= 79520.05

y_{LV03} = Y + 600000 = 679520.05

E_{LV95} = Y + 2600000 = 2679520.05

$$X = \frac{R}{2} \cdot \ln\left(\frac{1 + \sin\bar{b}}{1 - \sin\bar{b}}\right)$$

= 12273.44

x_{LV03} = X + 200000 = 212273.44

N_{LV95} = X + 1200000 = 1212273.44

3.3 Swiss projection coordinates (y, x) ⇒ ellipsoidal coordinates (λ, φ) (rigorous formulas)

Again the point Rigi was used as an example (in LV95): E = 2679520.05 N = 1212273.44

a) projection plane (y, x) ⇒ sphere ($\bar{b}$, $\bar{l}$)

$Y = y_{LV03} - 600'000$	$Y = E_{LV95} - 2'600'000$	= 79520.05
$X = x_{LV03} - 200'000$	$X = N_{LV95} - 1'200'000$	= 12273.44

$$\bar{l} = \frac{Y}{R}$$ 0.01246627136 rad

$$\bar{b} = 2 \cdot \left[\arctan\left(e^{\frac{X}{R}} \right) - \frac{\pi}{4} \right]$$ 0.00192409259 rad

b) pseudo-equator system ($\bar{b}$, $\bar{l}$) ⇒ equator system (b, l)

$$b = \arcsin\left(\cos b_0 \cdot \sin\bar{b} + \sin b_0 \cdot \cos\bar{b} \cdot \cos\bar{l} \right)$$ = 0.820535226 rad

$$l = \arctan\left(\frac{\sin\bar{l}}{\cos b_0 \cdot \cos\bar{l} - \sin b_0 \cdot \tan\bar{b}} \right)$$ = 0.0182840649 rad

c) sphere (b, l) ⇒ Bessel ellipsoid (φ, λ)

$$\lambda = \lambda_0 + \frac{l}{\alpha}$$ = 0.148115967 rad

= 8° 29' 11.111272"

$$S = \ln\tan\left(\frac{\pi}{4} + \frac{\varphi}{2} \right) = \frac{1}{\alpha}\left[\ln\tan\left(\frac{\pi}{4} + \frac{b}{2} \right) - K \right] + E \cdot \ln\tan\left(\frac{\pi}{4} + \frac{\arcsin(E \cdot \sin\varphi)}{2} \right)$$

$$\varphi = 2\arctan\left(e^{S} \right) - \frac{\pi}{2}$$

The equations for φ and also for S have to be resolved by **iteration**. As a starting value we propose φ = b.

The iteration steps give the following results:

0. step	S = 0	φ = 0.820535226
1. step	S = 0.933114264192610	φ = 0.821315364725524
2. step	S = 0.933117825679560	φ = 0.821317791017021
3. step	S = 0.933117836751434	φ = 0.821317798559814
4. step	S = 0.933117836785854	φ = 0.821317798583263
5. step	S = 0.933117836785961	φ = 0.821317798583336
6. step	S = 0.933117836785961	φ = 0.821317798583336
		φ = 47° 03' 28.956592"

3.4 Swiss projection coordinates ⇒ ellipsoidal coordinates (φ, λ) (approximate formulas)

simplified from: [Bolliger 1967]

Notation and units

φ, λ = ellipsoidal latitude and longitude relative to Greenwich **in [10000 "]**
Y, X = civilian projection coordinates **in [1000 km]**
y, x = official projection coordinates in LV03 **in [1000 km]**
E, N = official projection coordinates in LV95 **in [1000 km]**

Calculation

$Y = y_{LV03} - 0.6$ $X = x_{LV03} - 0.2$ resp.
$Y = E_{LV95} - 2.6$ $X = N_{LV95} - 1.2$

$\lambda = 2.67825 + a1*Y + a3*Y^3 + a5*Y^5$ with

a1 =	+ 4.729 730 56		a3 =	- 0.044 270		a5 =	+ 0.000 96	
	+ 0.792 571 4	* X		- 0.025 50	* X			
	+ 0.132 812	* X^2		- 0.009 6	* X^2			
	+ 0.025 50	* X^3						
	+ 0.004 8	* X^4						

$\varphi = 16.902866 + p0 + p2*Y^2 + p4*Y^4$ with

p0 =	0		p2 =	- 0.271 353 79		p4 =	+ 0.002 442	
	+ 3.238 648 77	* X		- 0.045 044 2	* X		+ 0.001 32	* X
	- 0.002 548 6	* X^2		- 0.007 553	* X^2			
	- 0.013 245	* X^3		- 0.001 46	* X^3			
	+ 0.000 048	* X^4						

approximation error (for IYI < 0.2 and IXI < 0.1):

approximation to 3rd degree:	Δλ <	0.16"	and	Δφ <	0.04"
approximation to 5th degree:	Δλ <	0.00014"	and	Δφ <	0.00004"

To control the calculation, the example (point Rigi) of the preceding chapter can be used. Further approximate formulas and examples can be found in [Bolliger 1967].

3.5 Ellipsoidal coordinates (λ, φ) ⇒ Swiss projection coordinates (approximate formulas)

simplified from: [Bolliger 1967]

Notation and units

φ, λ = ellipsoidal latitude and longitude relative to Greenwich **in [10'000 "]**
Y, X = civilian projection coordinates **in [1000 km]**
y, x = official (military) projection coordinates in LV03 **in [1000 km]**
E, N = projection coordinates in LV95 **in [1000 km]**

auxiliary values:

$\Phi = \varphi - 16.902866''$

$\Lambda = \lambda - 2.67825''$

Calculation

$Y = y1 * \Lambda + y3 * \Lambda^3 + y5 * \Lambda^5$ with

$$y1 = +0.211\,428\,533\,9 - 0.010\,939\,608 * \Phi - 0.000\,002\,658 * \Phi^2 - 0.000\,008\,53 * \Phi^3$$

$$y3 = -0.000\,044\,232\,7 + 0.000\,004\,291 * \Phi - 0.000\,000\,309 * \Phi^2$$

$$y5 = +0.000\,000\,019\,7$$

$X = x0 + x2 * \Lambda^2 + x4 * \Lambda^4$ with

$$x0 = 0 + 0.308\,770\,746\,3 * \Phi + 0.000\,075\,028 * \Phi^2 + 0.000\,120\,435 * \Phi^3 + 0 * \Phi^4 + 0.000\,000\,07 * \Phi^5$$

$$x2 = +0.003\,745\,408\,9 - 0.000\,193\,792\,7 * \Phi + 0.000\,004\,340 * \Phi^2 - 0.000\,000\,376 * \Phi^3$$

$$x4 = -0.000\,000\,734\,6 + 0.000\,000\,144\,4 * \Phi$$

$y_{LV03} = Y + 0.6$ $x_{LV03} = X + 0.2$ resp.
$E_{LV95} = Y + 2.6$ $N_{LV95} = X + 1.2$

approximation error (for $|\Lambda| < 1.0$ and $|\Phi| < 0.316$):

approximation to 3rd degree:	ΔY < 1.2 m	and	ΔX < 0.75 m	
approximation to 5th degree:	ΔY < 0.001 m	and	ΔX < 0.0007 m	

To control the calculation, the example (point Rigi) of the preceding chapter can be used. Further approximate formulas and examples can be found in [Bolliger 1967].

3.6 Formulas for meridian convergence and for scale distortion

The distortions, caused by the projection, can be described completely by the **meridian convergence μ** (angle between the ellipsoidal north direction and the grid north direction of the projection) and the **scale distortion m** (relationship of an infinitesimally small line in the projection and on the ellipsoid):

meridian convergence:

$$\mu = \arctan \frac{\sin b_0 \cdot \sin l}{\cos b_0 \cdot \cos b + \sin b_0 \cdot \sin b \cdot \cos l}$$

approximate formula:

$$\mu = 10.668 \cdot 10^{-6} \cdot Y + 1.788 \cdot 10^{-12} \cdot Y \cdot X - 0.14 \cdot 10^{-18} \cdot Y^3$$

Y and X denote the projection coordinates in the civilian system in [m]. The meridian convergence μ is obtained in Grads (Gons).

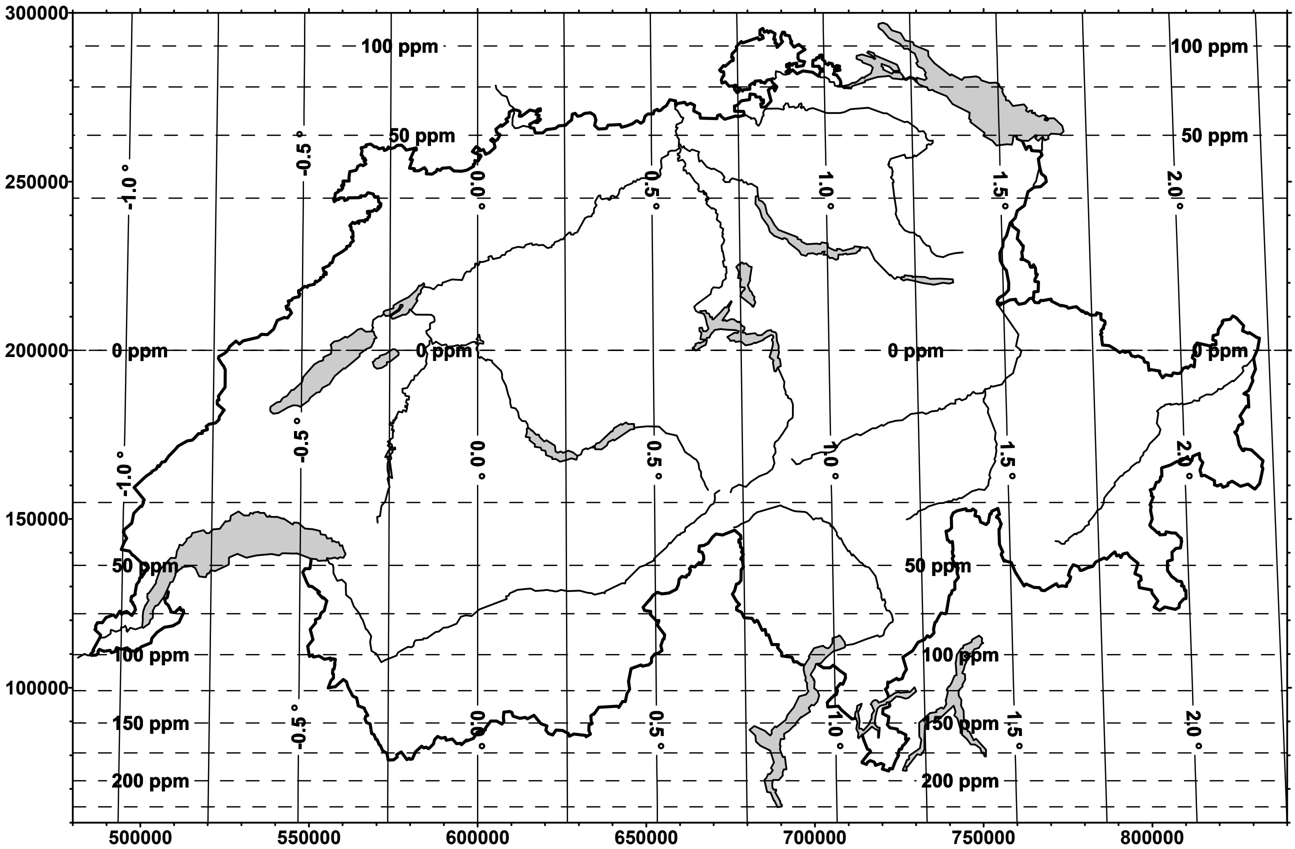
Scale distortion (main term):

$$m = \frac{s_{proj}}{s_{ell}} = \alpha \cdot \frac{R}{R_N} \cdot \frac{\cos b}{\cos \varphi \cdot \cos \overline{b}}$$

approximate formula:

$$m = 1 + \frac{X^2}{2R^2}$$

Example: Point Rigi (E = 2679520.05, N = 1212273.44)
from geographic coordinates: μ = 0.8499955 gon, m = 1.000001852
from approximate formulas: μ = 0.8499946 gon, m = 1.000001851



Representation of the meridian convergence (in degrees) and of the scale distortion (dashed, in ppm)

4 Approximate formulas for the transformation between Swiss projection coordinates andWGS84

4.1 Approximate formulas for the transformation between Swiss projection coordinates andWGS84

(Precision in the order of 1 metre)

After: [H. Dupraz, Transformation approchée de coordonnées WGS84 en coordonnées nationales suisses, IGEO-TOPO, EPFL, 1992]

The parameters were re-determined by U. Marti (May 1999). In addition, the units were changed so that the parameters are comparable to the values published in [Bolliger 1967].

1. Convert the ellipsoidal latitudes φ and longitudes λ into arcseconds ["]
2. Calculate the auxiliary values (differences of latitude and longitude relative to Bern in the unit [10000"]):

$$\varphi' = (\varphi - 169028.66\,'')/10000$$
$$\lambda' = (\lambda - 26782.5\,'')/10000$$

3. Calculate projection coordinates in LV95 (E, N, h) or in LV03 (y, x, h)

$$E\,[m] = 2600072.37 + 211455.93 \cdot \lambda' - 10938.51 \cdot \lambda' \cdot \varphi' - 0.36 \cdot \lambda' \cdot \varphi'^2 - 44.54 \cdot \lambda'^3$$

$$y\,[m] = E - 2000000.00$$

$$N\,[m] = 1200147.07 + 308807.95 \cdot \varphi' + 3745.25 \cdot \lambda'^2 + 76.63 \cdot \varphi'^2 - 194.56 \cdot \lambda'^2 \cdot \varphi' + 119.79 \cdot \varphi'^3$$

$$x\,[m] = N - 1000000.00$$

$$h_{CH}\,[m] = h_{WGS} - 49.55 + 2.73 \cdot \lambda' + 6.94 \cdot \varphi'$$

4. Numerical example

given:	φ = 46° 02' 38.87"	λ = 8° 43' 49.79"	h_{WGS} = 650.60 m
→	φ' = -0.326979	λ' = 0.464729	
→ LV95	E = 2 699 999.76 m	N = 1 099 999.97 m	h_{CH} = 600.05 m
→ LV03	y = 699 999.76 m	x = 99 999.97 m	h_{CH} = 600.05 m
Reference:	y = 700 000.0 m	x = 100 000.0 m	h_{CH} = 600 m

The precision of the approximate formulas is better than 1 metre in position and 0.5 metres in height everywhere in Switzerland.

Remark on the heights:
In these formulas, one is supposed to work with ellipsoidal heights as obtained by GPS measurements. If 'heights above sea level' are used, the heights are the same in both systems on the 1 metre level. Therefore, no transformation is necessary.

4.2 Approximate formulas for the direct transformation of Swiss projection coordinates to ellipsoidal WGS84 coordinates

(Precision in the order of 0.1")

These formulas were derived by U. Marti in May 1999, based on the formulas in [Bolliger, 1967]

1. Convert the projection coordinates E (easting) and N (northing) in LV95 (or y / x in LV03) into the civilian system (Bern = 0 / 0) and express in the unit [1000 km]:

$$E' = (E - 2600000\ \text{m})/1000000 = (y - 600000\ \text{m})/1000000$$
$$N' = (N - 1200000\ \text{m})/1000000 = (x - 200000\ \text{m})/1000000$$

2. Calculate longitude λ and latitude φ in the unit [10000"]:

$$\lambda' = 2.6779094 + 4.728982 \cdot y' + 0.791484 \cdot y' \cdot x' + 0.1306 \cdot y' \cdot x'^2 - 0.0436 \cdot y'^3$$

$$\varphi' = 16.9023892 + 3.238272 \cdot x' - 0.270978 \cdot y'^2 - 0.002528 \cdot x'^2 - 0.0447 \cdot y'^2 \cdot x' - 0.0140 \cdot x'^3$$

$$h_{WGS}\ [\text{m}] = h_{CH} + 49.55 - 12.60 \cdot y' - 22.64 \cdot x'$$

3. Convert longitude and latitude to the unit [°]

$$\lambda = \lambda' \cdot 100 / 36$$
$$\varphi = \varphi' \cdot 100 / 36$$

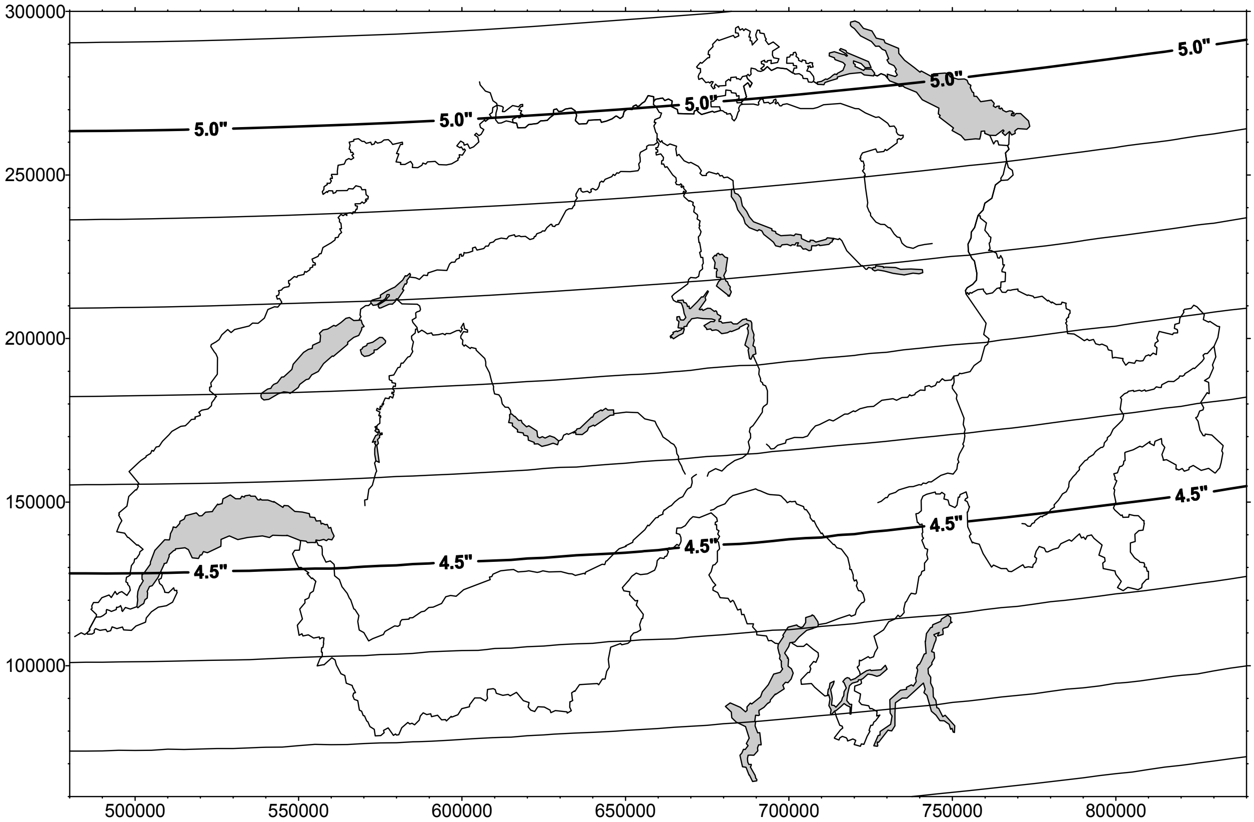
4. Numerical example

given:	E = 2 700 000 m	N = 1 100 000 m	h_{CH} = 600 m
→	y' = 0.1	x' = -0.1	
→	λ' = 3.14297976	φ' = 16.57588564	h_{WGS} = 650.55 m
→	λ = 8° 43' 49.80"	φ = 46° 02' 38.86"	
Reference:	λ = 8° 43' 49.79"	φ = 46° 02' 38.87"	h = 650.60 m

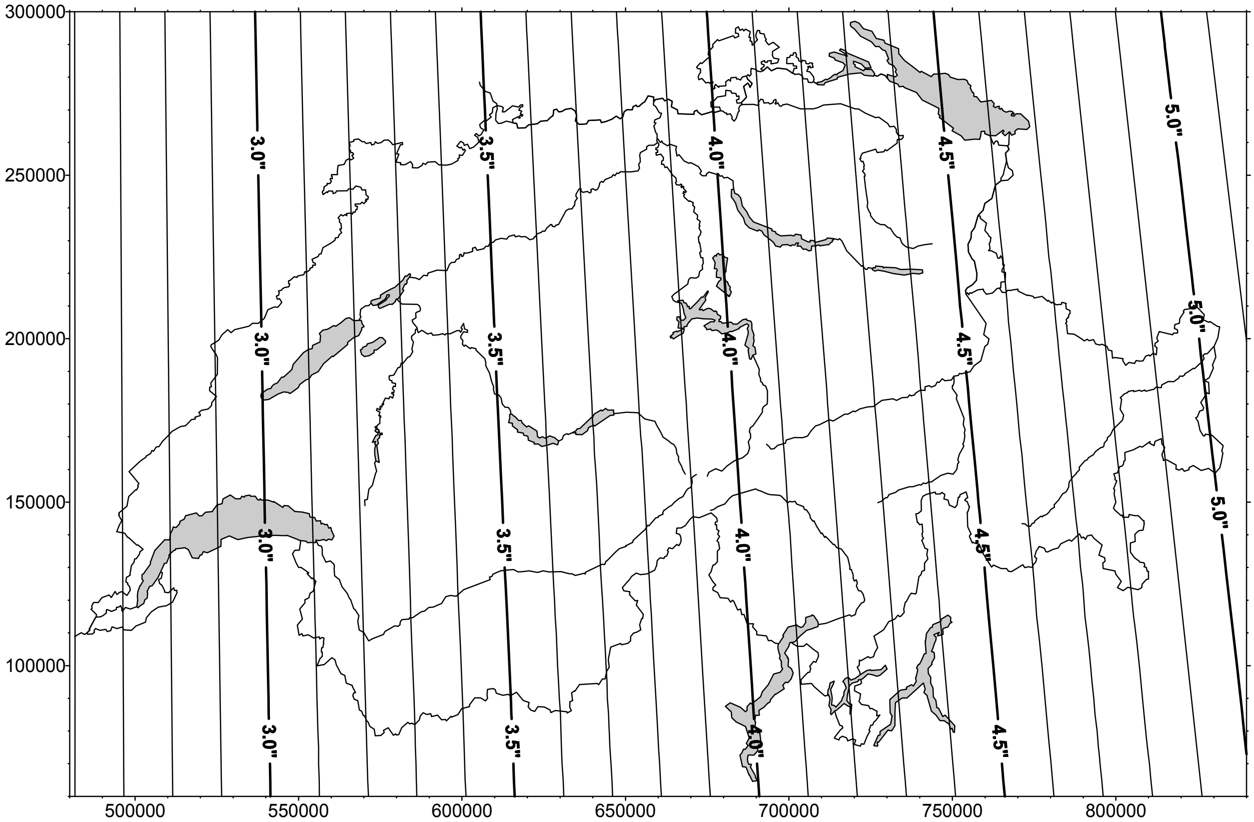
The precision of the approximate formulas is better than 0.12" in longitude, 0.08" in latitude and 0.5 metres in height everywhere in Switzerland.

Remark on the heights:
In these formulas one is supposed to work with ellipsoidal heights as obtained by GPS measurements. If 'heights above sea level' are used, the heights are the same in both systems on the 1 metre level. Therefore, no transformation is necessary.

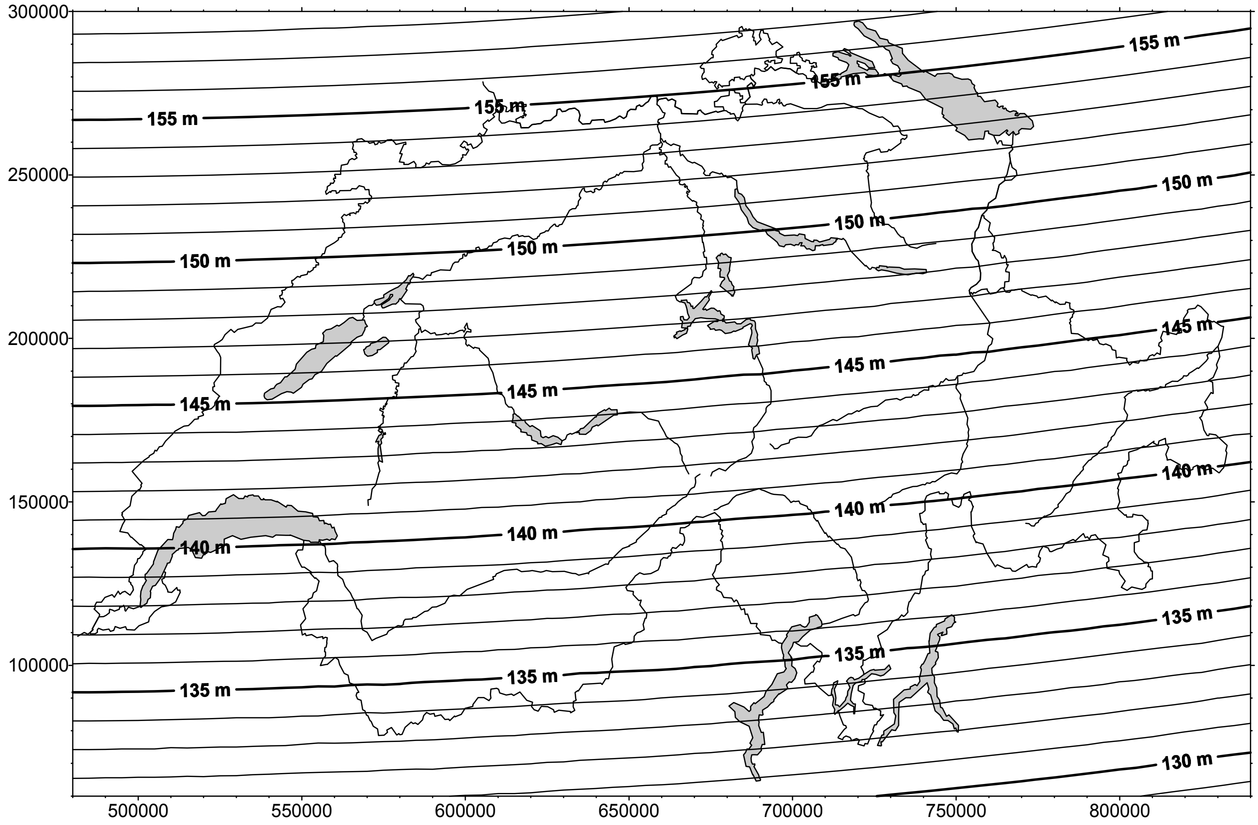
5 Diagrams of the differences between CH1903/CH1903+ and ETRS89/WGS84



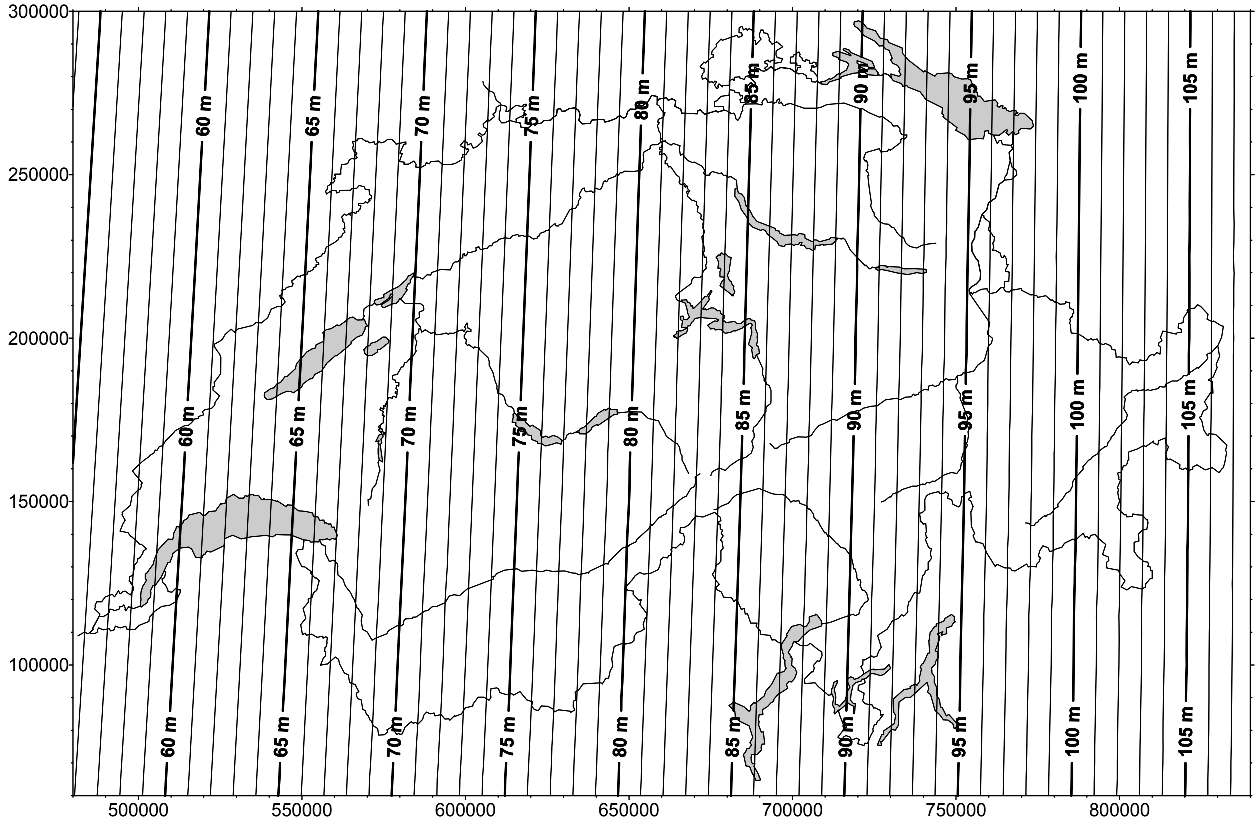
Differences in latitude: CH1903 minus ETRS89 (in arc seconds)



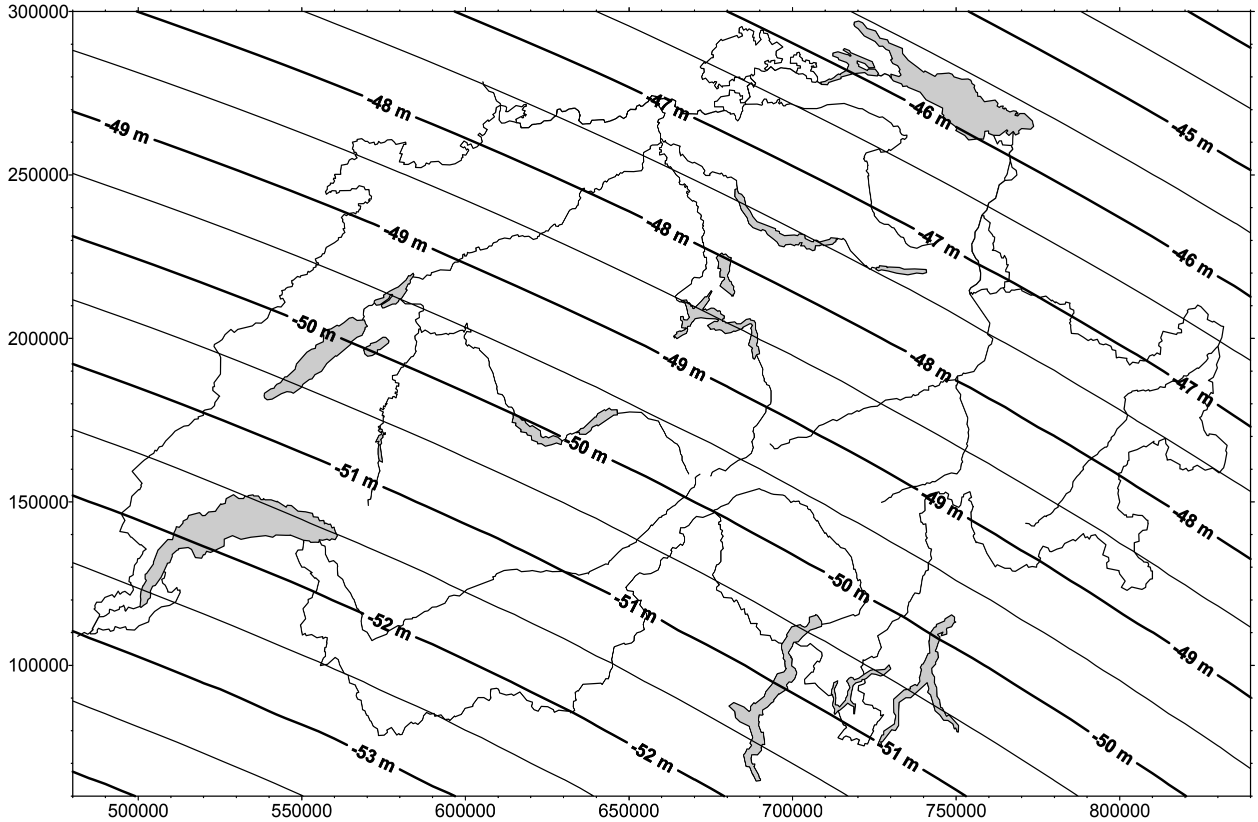
Differences in longitude: CH1903 minus ETRS89 (in arc seconds)



Differences in latitude: CH1903 minus ETRS89 (converted to metres)



Differences in longitude: CH1903 minus ETRS89 (converted to metres)



Differences in ellipsoidal height: CH1903 minus ETRS89 [in metres]

6 Summary of transformations

6.1 from LV03/LN02 to ETRS89

For a strict transformation from LV03 projection coordinates with heights in LN02 to ETRS89 the following steps are necessary:

1. Transformation of the LN02 heights to orthometric heights LHN95 by means of the HTRANS grids (can be omitted for height transformations on the 1-metre level or if heights are of no importance)
2. Transformation of orthometric heights LHN95 to ellipsoidal heights using the geoid model CHGeo2004 (can be omitted if height is of no importance; on the 1-metre level other geoid models may be used)
3. Transformation of LV03 to LV95 by using the Fineltra algorithm (data set CHENyx06) or a grid approximation of the differences (can be omitted on the 1-metre level)
4. Conversion to ellipsoidal coordinates CH1903+ on the Bessel ellipsoid using the inverse projection formulas of chapter 3.3.
5. Conversion to geocentric Cartesian coordinates (datum CH1903+) by using the formulas of chapter 2.1 and the parameters of the Bessel ellipsoid (if the height is unknown or of no importance, an approximate value can be used - or even set to 0, which does not affect the result significantly).
6. Datum transformation from CH1903+ to ETRS89 (CHTRS95) by using the parameters of chapter 1.4.
7. Calculation of ellipsoidal coordinates in ETRS89 by using the formulas of chapter 2.2 and the parameters of the GRS80 ellipsoid.
8. The further treatment of the resulting ETRS89 ellipsoidal coordinates (projection or height transformation) is not discussed in this document

On the 1-metre level (inside of Switzerland) step 4 can be replaced by the approximate formulas of chapter 3.4 or steps 4 to 7 can be approximated by the formulas of chapter 0.

6.2 from ETRS89 to LV03/LN02

For a strict transformation from ETRS89 ellipsoidal coordinates with ellipsoidal heights to LV03/LN02 the following steps are necessary:

1. Conversion to geocentric Cartesian coordinates (datum ETRS89) by using the formulas of chapter 2.1 and the parameters of the GRS80 ellipsoid (if the height is unknown or of no importance, an approximate value can be used - or even set to 0, which does not affect the result significantly).
2. Datum transformation from ETRS89 (CHTRS95) to CH1903+ by using the parameters of chapter 1.4.
3. Calculation of ellipsoidal coordinates in CH1903+ by using the formulas of chapter 2.2 and the parameters of the Bessel ellipsoid.
4. Calculation of LV95 coordinates (with ellipsoidal heights) by using the projection formulas of chapter 3.2.
5. Transformation of LV95 to LV03 by using the Fineltra algorithm (data set CHENyx06) or a grid approximation of the differences (can be omitted on the 1-metre level)
6. Transformation of ellipsoidal heights to orthometric heights LHN95 using the geoid model CHGeo2004 (can be omitted if height is of no importance; on the 1-metre level other geoid models may be used)
7. Transformation of the LHN95 heights to LN02 by means of the HTRANS grids (can be omitted for height transformations on the 1-metre level or if heights are of no importance)

On the 1-metre level (inside of Switzerland), step 4 can be replaced by the approximate formulas of chapter 3.5 or steps 1 to 4 can be approximated by the formulas of chapter **Fehler! Verweisquelle konnte nicht gefunden werden.**.

7 Numerical example

7.1 Coordinate transformation LV03/LN02 ⇒ ETRS89

As input for this example, the EUREF points of Switzerland are used. All calculations have been performed with the swisstopo programs REFRAME and GEOREF. Small differences (<1 mm) in the results may be caused due to rounding effects.

Swiss projection coordinates LV03 with heights LN02

Zimmerwald	602030.680	191775.030	897.915
Chrischona	617306.300	268507.300	456.064
Pfaender	776668.105	265372.681	1042.624
La Givrine	497313.292	145625.438	1207.434
Monte Generoso	722758.810	87649.670	1636.600

⇒ FINELTRA-transformation with CHENyx06 and height transformation with HTRANS ⇒

Swiss projection coordinates LV95 with orthometric heights LHN95

Zimmerwald	2602030.740	1191775.030	897.906
Chrischona	2617306.920	1268507.870	455.915
Pfaender	2776668.590	1265372.250	1042.528
La Givrine	2497312.650	1145626.140	1207.473
Monte Generoso	2722759.060	1087648.190	1636.794

⇒ Calculation and addition of the geoid model (CHGeo2004) ⇒

Swiss projection coordinates LV95 with ellipsoidal heights and geoid undulation

Zimmerwald	2602030.740	1191775.030	897.361	-0.5454
Chrischona	2617306.920	1268507.870	457.138	1.2233
Pfaender	2776668.590	1265372.250	1043.616	1.0880
La Givrine	2497312.650	1145626.140	1206.367	-1.1060
Monte Generoso	2722759.060	1087648.190	1634.472	-2.3227

⇒ Conversion to ellipsoidal coordinates

ellipsoidal coordinates and heights in CH1903+

Zimmerwald	7 27 58.416328	46 52 42.269284	897.361
Chrischona	7 40 10.574820	47 34 6.404965	457.138
Pfaender	9 47 8.465989	47 31 0.092644	1043.616
La Givrine	6 6 9.983811	46 27 19.272743	1206.367
Monte Generoso	9 1 20.606368	45 55 49.707052	1634.472

⇒ Conversion to geocentric-cartesian coordinates

Geocentric-Cartesian coordinates in CH1903+

Zimmerwald	4330616.737	567539.766	4632721.664
Chrischona	4272473.562	575353.239	4684498.293
Pfaender	4252889.174	733507.303	4681046.757
La Givrine	4377121.142	467993.592	4600671.934
Monte Generoso	4389483.221	696984.352	4560589.600

⇒ datum transformation from CH1903+ to ETRS89 ⇒

Geocentric-Cartesian coordinates in ETRS89 / CHTRS95

Zimmerwald	4331291.111	567554.822	4633127.010
Chrischona	4273147.936	575368.294	4684903.639
Pfaender	4253563.548	733522.359	4681452.103
La Givrine	4377795.516	468008.648	4601077.280
Monte Generoso	4390157.595	696999.408	4560994.946

⇒ conversion to ellipsoidal coordinates

ellipsoidal coordinates and ellipsoidal heights in ETRS89

Zimmerwald	7 27 54.983506	46 52 37.540562	947.149
Chrischona	7 40 6.983077	47 34 1.385301	504.935
Pfaender	9 47 3.697723	47 30 55.172797	1089.372
La Givrine	6 6 7.326361	46 27 14.690021	1258.274
Monte Generoso	9 1 16.389053	45 55 45.438020	1685.027

7.2 Coordinate transformation ETRS89 ⇒ LV03/LN02

To test this calculation, the same points and numbers as in the example of chapter 7.1 can be used - just in reverse order.

8 References

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